Coloring of Geometrical Configurations

19 July, 1995

Jaroslav Nesetril

The Charles University, Prague

O. Abstract

We consider the following problem: Given natural numbers p, k and a finite plane set X, does there exist a plane set Y with the following property: For every coloring of all p-element subsets of Y by k colors there exists a monochromatic subset X' that is "convex- equivalent" to X and such that no other point of Y lies in the convex hull of X'. We give an affirmative answer for $p = 1$, and this may be seen as an induced and hole-preserving version of a theorem of Erdös and Szekeres theorem. For $p = 2$ the problem has a negative solution.

Secondly we survey results on colorings under a hereditary density assumption (known also as Pisier-type theorems). These are questions of the following type: Let $0 < c < 1$ be a real number, and let X be a (possibly infinite) set such that every finite subset Y of X contains $c|Y|$ independent points. Is then X a finite union of independent sets? We present motivation and geometrical examples of such statements for various notions of independency.

The research described here is joint work with P.Valtr and with P.Erdös and V.Rödl, respectively.