An Info Commercial

20 November, 1995

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O. Abstract

Not available. (See also the enclosed whitepapers *Partitioning Permutations into Increasing and Decreasing Subsequences*, *The 2-Pebbling Property and a Conjecture of Graham's*, and *On Pebbling Graphs*.

I. Forbidden Subsequences

Let us write permutations in *linear order*, as

$$(1534627) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 3 & 4 & 6 & 2 & 7 \end{pmatrix}$$

and suppose we wish to decompose a permutation into a list of r strictly increasing and s strictly decreasing (possible empty) permutations. An important result then is that $\forall (r, s)$, there is a finite list of forbidden subsequences. For example, $(1, 1)$ must avoid 2143 and 3412.

To prove the above, Wang, Snevily, et. al. considered permutation graphs whose edges are given by (i, j) such that $(\pi(i) - \pi(j))/(i - j) < 0$. It turns out that forbidden subsequences in the permutation correspond to cliques in the graph, and permutation graphs are perfect, so $\chi(G) = \omega(G)$ and $\chi(H) = \omega(H)$ $\forall$ subgraphs H of G.

II. Pebbling and Graham's Conjecture

See the enclosed papers on this subject. Since receiving these, Matt Hudelson also got some results on the optimal sanding numbers of graphs.

III. Pattern avoidance

(an appropriate story of how *not* to go about doing research...)

XX cannot be avoided in a binary sequence of length 4 or greater – however, it is 3-avoidable (i.e., in ternary sequences). What about XXX? Or even XXe, where e is the first letter of X? (Such words are said to be cube-free, or strongly cube-free, respectively). In fact, there exist infinitely long strongly cube-free binary words. Let $h(0) = 01$, and $h(1) = 10$. Then $h^2(0) = h(h(0)), \ldots$, and the words defined by $w_n = h^n(0)\overline{h^n(0)}$ (where the overbar denotes binary complement) give rise to a limiting word $w_\infty = \lim_{n\to\infty} w_n$ which is strongly cube-free (1912, Thue).

More generally, given a pattern of n distinct letters, it is avoidable iff it does not occur in Zimm's word Z_n, which is defined by $Z_1 = 1$, $Z_2 = 121$, $Z_3 = 1213121$, and in general, $Z_n = Z_{n-1} n Z_{n-1}$.