Orbifolds

23 October, 1995 ?

Someone

Universität Bielefeld

O. Abstract

Not available

I. Naming symmetry groups

Different naming schemes for symmetry groups lead to difficulties. Spherical tilings have some infinite families; hyperbolic tilings have infinitely many types! So it would be nice to have some better naming schemes. Macbeath (1967) made a "group signature" but it's ugly. Conway's oribfold notation (1992, *A Visit to Symmetry Land*, Proceedings of the London Mathematical Society) is much nicer.

II. Orbifold notation

Given a compact 2-dimensional manifold M, consider a discrete set of identified points ("cone" oints in the interior, or "corner" points in the boundary) $S = \{(x_1, v_1), \ldots, (x_k, v_k)\} \subset M \times N$. For example, any crystallographic group Γ gives rise to an orbifold M/Γ. Translations on the torus ($p1$) are written 0. On the sphere with some identified points, ($p2$), we write 2222. The tetrahedron becomes 332. More generally, on a 2−surface we have orientable orbifolds, the sphere with q holes and h handles; and non-orientable orbifolds, the sphere with q holes and k cross-caps. Then we write the orbifold symbol

$$\underbrace{0 \cdots\cdots\cdots}_{\text{handles}} \underbrace{p_1 \cdots\cdots\cdots}_{\text{cone points}} \underbrace{\star q_1 \cdots\cdots\cdots}_{\text{corner points}} \underbrace{x \cdots\cdots\cdots}_{\text{cross caps}}$$

When n in the spherical tiling is replaced by ∞, one gets a "frieze" tiling, or "strip" tiling. Two orbifolds are "of the same form" if they are equal up to a permutation of $N/\{0,1\}$. For example, $\star 632$, $\star 532$, $\star 432$, and even $\star 96, 27, 11$ are all of the same form. In this case there is a one-to-one correspondence between the set of periodic tilings with these orbifolds as their symmetry groups.

Showing this encounters some difficulties for rotations (degree 2) about an edge, but we can just add a vertex to the center of that edge, and then increase its degree (this was illustrated with some nice diagrams). Conversely, if one sends a vertex of high degree to one of degree 2, simply remove that vertex, leaving just the edge.

Software is available to generate these tilings (though right now it's just for SGI workstations) at the URL

`ftp://ftp.uni-bielefeld.de/pub/math/tiling/funtile`